Functions

1 Critical and Stationary Points

A critical point exists where the derivative is zero or undefined.

Stationary points can be:

- maximum turning points;
- minimum turning points;
- horizontal points of inflection.

2 Derivative Tests

First derivative test

The first derivative test is the nature table method used in Higher.

Second derivative test

If a stationary point exists at \( x = a \) then the second derivative test is to calculate \( f''(a) \).

- If \( f''(a) < 0 \) then the point is a maximum turning point.
- If \( f''(a) > 0 \) then the point is a minimum turning point.

The second derivative test may not always work; if \( f''(a) = 0 \) then you should revert to the first derivative test.

Non-horizontal points of inflection

A non-horizontal point of inflection exists at \( x = a \) when \( f''(a) = 0 \) but \( f'(a) \neq 0 \) and \( f''(x) \) changes sign at \( x = a \), i.e. the curve changes from being concave up to concave down or vice versa.
3 Curve Sketching

1. Identify $x$-axis and $y$-axis intercepts.
2. Identify turning points and their nature.
3. Consider the behaviour as $x \to \pm \infty$.
   This could be a horizontal asymptote or a slant asymptote (the equation of which is the polynomial part after algebraic long division).
4. Check where $y$ is undefined (this means a vertical asymptote exists).
5. Sketch the curve showing asymptotes, turning points and $x$- and $y$-axis intercepts.

Transformations of curves

All transformations from Higher should be known.

The modulus function

To sketch the modulus of a function, take any part of the curve which is below the $x$-axis and reflect it in the $x$-axis.