Read carefully

1. Calculators may be used in this paper.

2. There are five Sections in this paper.
   - Section A assesses the compulsory units Mathematics 1 and 2
   - Section B assesses the optional unit Mathematics 3
   - Section C assesses the optional unit Statistics 1
   - Section D assesses the optional unit Numerical Analysis 1
   - Section E assesses the optional unit Mechanics 1.

Candidates must attempt Section A (Mathematics 1 and 2) and one of the following Sections:
   - Section B (Mathematics 3)
   - Section C (Statistics 1)
   - Section D (Numerical Analysis 1)
   - Section E (Mechanics 1).

3. Candidates must use a separate answer book for each Section. Take care to show clearly the optional section chosen. On the front of the answer book, in the top right hand corner, write B, C, D or E.

4. A booklet of Mathematical Formulae and Statistical Tables is supplied for all candidates. It contains Numerical Analysis formulae and Statistical formulae and tables.

5. Full credit will be given only where the solution contains appropriate working.
Section A (Mathematics 1 and 2)

All candidates should attempt this Section.

Answer all the questions.

A1. Use Gaussian elimination to solve the following system of equations

\[
\begin{align*}
  x + y + 3z &= 2 \\
  2x + y + z &= 2 \\
  3x + 2y + 5z &= 5.
\end{align*}
\]

A2. Verify that \( i \) is a solution of \( z^4 + 4z^3 + 3z^2 + 4z + 2 = 0 \).

 Hence find all the solutions.

A3. A curve is defined by the parametric equations

\[
x = t^2 + t - 1, \quad y = 2t^2 - t + 2
\]

for all \( t \). Show that the point \( A \) \((-1, 5)\) lies on the curve and obtain an equation of the tangent to the curve at the point \( A \).

A4. (a) Given that \( f(x) = \sqrt{x} e^{-x} \), \( x \geq 0 \), obtain and simplify \( f'(x) \).

(b) Given \( y = (x + 1)^2 (x + 2)^{-4} \) and \( x > 0 \), use logarithmic differentiation to show that \( \frac{dy}{dx} \) can be expressed in the form \( \left( \frac{a}{x+1} + \frac{b}{x+2} \right) y \), stating the values of the constants \( a \) and \( b \).

A5. Use integration by parts to evaluate \( \int_0^1 \ln(1 + x) \, dx \).

A6. Use the substitution \( x + 2 = 2 \tan \theta \) to obtain \( \int \frac{1}{x^2 + 4x + 8} \, dx \).

A7. Prove by induction that \( 4^n - 1 \) is divisible by 3 for all positive integers \( n \).
A8. Express \( \frac{x^2}{(x+1)^2} \) in the form \( A + \frac{B}{x+1} + \frac{C}{(x+1)^2} \), \( x \neq -1 \), stating the values of the constants \( A \), \( B \) and \( C \).

A curve is defined by \( y = \frac{x^2}{(x+1)^2} \), \( x \neq -1 \).

(i) Write down equations for its asymptotes. 2
(ii) Find the stationary point and justify its nature. 4
(iii) Sketch the curve showing clearly the features found in \( (i) \) and \( (ii) \). 2

A9. Functions \( x(t) \) and \( y(t) \) satisfy
\[
\frac{dx}{dt} = -x^2 y, \quad \frac{dy}{dt} = -xy^2.
\]
When \( t = 0 \), \( x = 1 \) and \( y = 2 \).

(a) Express \( \frac{dy}{dx} \) in terms of \( x \) and \( y \) and hence obtain \( y \) as a function of \( x \). 5

(b) Deduce that \( \frac{dx}{dt} = -2x^3 \) and obtain \( x \) as a function of \( t \) for \( t \geq 0 \). 5

A10. Define \( S_n(x) \) by
\[
S_n(x) = 1 + 2x + 3x^2 + \ldots + nx^{n-1},
\]
where \( n \) is a positive integer.

Express \( S_n(1) \) in terms of \( n \). 2

By considering \( (1 - x) S_n(x) \), show that
\[
S_n(x) = \frac{1 - x^n}{(1 - x)^2} - \frac{nx^n}{(1 - x)}, \quad x \neq 1.
\]

Obtain the value of \( \lim_{n \to \infty} \left( \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots + \frac{n}{3^{n-1}} + \frac{2}{3^n} \right) \). 3

[END OF SECTION A]

Candidates should now attempt ONE of the following

Section B (Mathematics 3) on Page four
Section C (Statistics 1) on Pages five and six
Section D (Numerical Analysis 1) on Pages seven and eight
Section E (Mechanics 1) on Pages nine and ten.

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[Turn over
Section B (Mathematics 3)

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

Answer all the questions.

Answer these questions in a separate answer book, showing clearly the section chosen.

B1. (a) Find an equation for the plane \( \pi_1 \) which contains the points \( A(1, 1, 0) \), \( B(3, 1, -1) \) and \( C(2, 0, -3) \).  

(b) Given that \( \pi_2 \) is the plane whose equation is \( x + 2y + z = 3 \), calculate the size of the acute angle between the planes \( \pi_1 \) and \( \pi_2 \).

B2. A matrix \( A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \). Prove by induction that

\[
A^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix},
\]

where \( n \) is any positive integer.

B3. Find the Maclaurin expansion of

\[
f(x) = \ln(\cos x), \quad 0 \leq x < \frac{\pi}{2},
\]

as far as the term in \( x^4 \).

B4. Write down the \( 2 \times 2 \) matrix \( A \) representing a reflection in the \( x \)-axis and the \( 2 \times 2 \) matrix \( B \) representing an anti-clockwise rotation of 30° about the origin.

Hence show that the image of a point \( (x, y) \) under the transformation \( A \) followed by the transformation \( B \) is \( \left( \frac{kx + y}{2}, \frac{x - ky}{2} \right) \), stating the value of \( k \).

B5. Find the general solution of the differential equation

\[
\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4 \cos x.
\]

Hence determine the solution which satisfies \( y(0) = 0 \) and \( y'(0) = 1 \).